## Exercise 23

Evaluate the integral.

$$\int_1^9 \sqrt{x}\,dx$$

---

**Solution**

According to part 2 of the fundamental theorem of calculus,

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where $F$ is an antiderivative of $f$. Use the power rule in reverse here: Bump up the exponent by 1 and divide by that exponent.

$$\begin{aligned}
\int_1^9 \sqrt{x}\,dx &= \int_1^9 x^{1/2}\,dx \\
&= \left(\frac{x^{3/2}}{\frac{3}{2}}\right)\Bigg|_1^9 \\
&= \frac{2}{3}\,(x^{3/2})\Big|_1^9 \\
&= \frac{2}{3}(9^{3/2} - 1^{3/2}) \\
&= \frac{2}{3}(3^3 - 1^3) \\
&= \frac{52}{3}
\end{aligned}$$